Int. J. Solids Structures Vol. 20, No. 11/12, pp. 1079–1090, 1984
Printed in the U.S.A.

0020-7683/84 $3.00+.00
Pergamon Press Ltd.

CONSOLIDATION AROUND A SPHERICAL HEAT SOURCE

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(*Received* 4 *May* 1983; *in revised form* 28 *December* 1983)

1. INTRODUCTION

If a heat source such as a canister of radioactive waste is buried in a saturated soil the source will cause a temperature rise in the soil. This temperature rise will cause both the soil pore water and the soil skeleton to expand. In general the volume increase of the pore water is greater than that of the voids in the soil skeleton and so the differential volume change leads to an increase in pore water pressure and a consequent reduction in effective stress. If this reduction in effective stress is too great the soil may fracture, leading to an increased rate of migration of pore water or even to a progressive failure, which ultimately reaches the surface. If the deposit is sufficiently permeable consolidation will occur and the excess pore water pressure, generated by the increase in temperature, will dissipate and reduce the severity of these effects.

In this paper an analytic solution is developed for the fundamental problem, a spherical heat source buried deep in a saturated thermo-elastic soil.

2. BASIC EQUATIONS

The equations for the one dimensional consolidation of a two phase elastic soil were first developed by Terzaghi [1]. These equations were generalised to include three dimensional effects by Biot [2] and were subsequently generalised to include the effects of anisotropy and visco-elasticity, Biot [3, 4].

A general treatment of the thermomechanical behaviour of a porous solid, containing several constituent pore fluids where both the matrix and the pore fluids are incompressible, has been developed by Bowen [5] using the modern theory of mixtures. Since in this paper attention is restricted to a simple two phase thermo-elastic soil, an alternative simple derivation of the governing equations, which allows for thermal expansion of both soil skeleton and pore water, and which is based on the effective stress principle, is preferred.

2.1. *Development of equations*

The equations governing the consolidation of a saturated thermo-elastic soil are:

2.1.1. *The equations of equilibrium.*

$$\boldsymbol{\partial}^T \boldsymbol{\sigma} = 0 \tag{1}$$

where

$$\boldsymbol{\partial} = \begin{bmatrix} \partial/\partial x & 0 & 0 & 0 & \partial/\partial z & \partial/\partial y \\ 0 & \partial/\partial y & 0 & \partial/\partial z & 0 & \partial/\partial x \\ 0 & 0 & \partial/\partial z & \partial/\partial y & \partial/\partial x & 0 \end{bmatrix}$$

$$\boldsymbol{\sigma}^T = (\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{zy}, \sigma_{zx}, \sigma_{xy})$$

and $\sigma_{xx}, \sigma_{yy}, \ldots, \sigma_{xy}$ denote the increase in total stress components over the initial equilibrium state (compressive stresses being taken as positive) and where changes to the density of soil, due to thermal expansion, have been neglected.

2.1.2. *The strain displacement relations.* Compressive strains are also considered positive and so:

$$\boldsymbol{\epsilon} = -\,\partial \mathbf{u} \tag{2}$$

where

$$\boldsymbol{\epsilon}^T = (\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}, \gamma_{zy}, \gamma_{zx}, \gamma_{xy})$$

$$\mathbf{u}^T = (u_x, u_y, u_z)$$

and u_x, u_y, u_z are the Cartesian components of deflection.

2.1.3. *Effective stress–strain temperature relation.* The effective stress–strain relations for an isotropic thermo-elastic soil may be written:

$$\begin{aligned}
\sigma_{xx} &= p + b'\theta + \lambda\epsilon_v + 2G\epsilon_{xx} \\
\sigma_{yy} &= p + b'\theta + \lambda\epsilon_v + 2G\epsilon_{yy} \\
\sigma_{zz} &= p + b'\theta + \lambda\epsilon_v + 2G\epsilon_{zz} \\
\sigma_{yz} &= G\gamma_{yz} \\
\sigma_{zx} &= G\gamma_{zx} \\
\sigma_{xy} &= G\gamma_{xy}
\end{aligned} \tag{3}$$

where

$$\epsilon_v = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} \quad \text{denotes the volume strain}$$

$$b' = \left(\lambda + \frac{2G}{3}\right) a'$$

and where λ, G are the Lame moduli of the soil skeleton and a' is the coefficient of volume expansion of the soil. Clearly if no structural changes occur during expansion a' will be identical to a_s, the coefficient of volume expansion of the skeleton material.

2.1.4. *The volume constraint equation.* Suppose that the volume change in the soil skeleton (due to stress) and the volume changes of the pore water (due to pressure) can be neglected. Then it is easy to establish that the volume outflow from a soil element must just match the decrease in volume of the element plus any increase in volume (due to an increase in temperature) of the constituents and so:

$$\int_0^t \nabla^T \mathbf{v} \, dt = \epsilon_v + a_u\theta \tag{4}$$

where n is the soil porosity and

$$a_u = a_s(1 - n) + a_w n$$

and a_w is the coefficient of expansion of the pore water.

2.1.5. *Darcy's law.* The flow of pore water in the soil is governed by Darcy's Law

$$\mathbf{v} = -\frac{k}{\gamma_w} \nabla p \tag{5}$$

where k is the coefficient of permeability of the soil and γ_w is the unit weight of water.

2.1.6. *Thermal energy balance.* In many applications mechanical contributions to energy balance may be neglected when compared to thermal contributions. In such cases the net rate of inflow of energy into an element of the material will be just balanced by increases in the internal energy of the pore water and the soil skeleton. The clays in which radioactive waste is to be buried are likely to be relatively impermeable, so that the velocity of the pore fluid may be expected to be small and consequently it will be permissible to ignore the convective component of heat transfer, so that:

$$-\nabla^T \mathbf{h} = m \frac{\partial \theta}{\partial t} - q \tag{6}$$

where q is the intensity of any distributed heat source; $m = n\rho_w c_w + (1 - n)\rho_s c_s$; $\mathbf{h}$ is the heat flux vector; and ρ_w, ρ_s are the densities of the pore water and the skeletal material, while c_w, c_s are their specific heats.

2.1.7. *Fourier's law of heat conduction.* The flow of heat in the soil is assumed to be governed by Fourier's Law

$$\mathbf{h} = -K \nabla \theta \tag{7}$$

where K is the coefficient of heat conduction.

2.2. *Equations for a homogeneous soil*

The above equations may be considerably simplified for a homogeneous material. If eqns (2) and (3) are substituted into eqn (1) it is found that

$$\begin{aligned} G\nabla^2 u_x - (\lambda + G)\frac{\partial \epsilon_v}{\partial x} &= \frac{\partial p}{\partial x} + b'\frac{\partial \theta}{\partial x} \\ G\nabla^2 u_y - (\lambda + G)\frac{\partial \epsilon_v}{\partial y} &= \frac{\partial p}{\partial y} + b'\frac{\partial \theta}{\partial y} \\ G\nabla^2 u_z - (\lambda + G)\frac{\partial \epsilon_v}{\partial z} &= \frac{\partial p}{\partial z} + b'\frac{\partial \theta}{\partial z}. \end{aligned} \tag{8}$$

When the volume constraint equation (4), and Darcy's Law (5) are combined it is found that

$$\epsilon_v + a_u\theta + \int_0^t \frac{k}{\gamma_w}\nabla^2 p \, dt = 0. \tag{9}$$

Finally, when the energy balance equation (6) and Fourier's Law (7) are combined

$$\kappa\nabla^2\theta = \frac{\partial \theta}{\partial t} - \frac{q}{m} \tag{10}$$

where

$$\kappa = \frac{K}{m}.$$

3. SOLUTION FOR A SPHERICAL SOURCE

Consider a rigid impermeable heat source, Fig. 1, which has been placed at a great depth below the surface of a homogeneous, saturated, thermo-elastic soil. Clearly because of the great depth of burial the problem exhibits spherical symmetry and so the

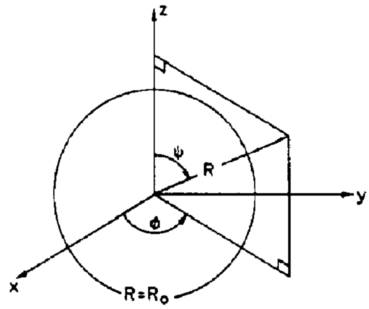


Fig. 1. Spherical heat source.

field quantities have the form:

$$
\begin{aligned}
p &= p(R, t) \\
\theta &= \theta(R, t) \\
(u_x, u_y, u_z) &= \frac{1}{R}(x, y, z)\, U_R(R, t).
\end{aligned} \tag{19}
$$

The displacements may thus be conveniently represented in the form

$$\mathbf{u} = \nabla\Omega \tag{20}$$

where

$$\Omega = \Omega(R, t).$$

3.1. *Temperature distribution*

The solution for the temperature distribution is most easily obtained by the application of a Laplace transform

$$\bar{\theta} = \int_0^\infty e^{-st}\,\theta(t)\,dt.$$

The heat equation together with the initial condition, that $\theta = 0$ when $t = 0$, then becomes

$$\nabla^2\bar{\theta} = \frac{s}{\kappa}\bar{\theta}. \tag{21a}$$

This equation must be solved subject to the boundary condition that

$$K\frac{\partial\bar{\theta}}{\partial R} = -\frac{\bar{Q}}{4\pi R_0^2} \quad \text{when} \quad R = R_0 \tag{21b}$$

where Q denotes the strength of the heat source.

For conditions of spherical symmetry equation (21a) becomes:

$$\frac{\partial^2}{\partial R^2}(R\bar{\theta}) = \frac{s}{\kappa}(R\bar{\theta}) \tag{21c}$$

and so

$$\bar{\theta} = \frac{s\bar{Q}}{4\pi KR}\bar{f}\,(s/\kappa) \tag{22a}$$

where

$$\bar{f} = \frac{1}{s}\frac{e^{-\xi(R-R_0)}}{1+\xi R_0}$$

and $\xi = \sqrt{(s/\kappa)}$ denotes the square root having a positive real part.

If the heat source is constant equation (22a) may be inverted to give

$$\theta = \frac{Q}{4\pi KR} f(\kappa, R, t) \tag{22b}$$

where

$$f = erfc(\beta) - e^{+\alpha^2 + \Delta R/R_0}\, erfc(\alpha + \beta)$$
$$\alpha^2 = \kappa t/R_0^2$$
$$\beta^2 = \Delta R^2/4\,\kappa t$$
$$\Delta R = R - R_0.$$

After a long period of time the temperature will reach a steady state distribution

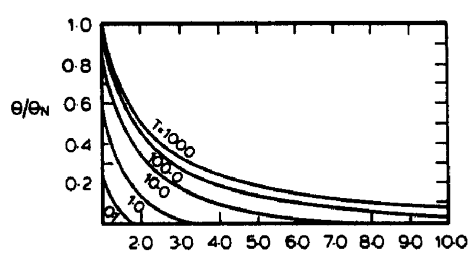
$$\theta = \theta_N \frac{R_0}{R} \tag{22c}$$

where $\theta_N = Q/4\pi KR_0$ is the final temperature on the surface of the sphere.

The distribution of temperature is shown in Figs. 2(a, b). The temperature has been normalised and is expressed as a fraction of the steady state temperature at the surface of the sphere. Figure 2(a) shows the spatial variation of normalised temperature at different values of the dimensionless time $T = \kappa t/R_0^2$ and Fig. 2(b) shows the variation of normalised temperature at particular locations as a function of time.

3.2. *Solution for a completely impermeable soil*

If the soil is relatively impermeable then the excess pore pressure generated by the increase in temperature will dissipate very slowly. It is consequently of interest to examine the limiting case of a completely impermeable soil ($k = 0$).



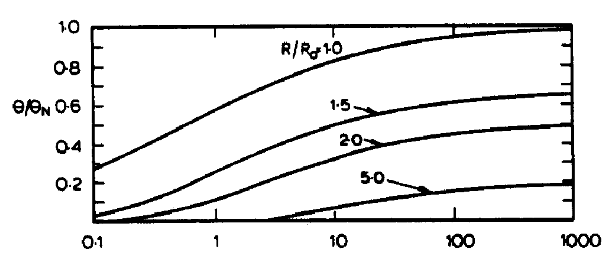


Fig. 2(a) Temperature isochrones for a spherical source. (b) Temperature distribution for a spherical source.

Under these circumstances it follows from eqn (4) that:

$$\epsilon_v = a_u \theta. \tag{23a}$$

It now follows from eqns (8) and (20) that:

$$\nabla[(\lambda + 2G)\epsilon_v + b'\theta + p] = 0$$

and thus recalling that the field quantities all vanish as $R \to \infty$

$$p = -[(\lambda + 2G)\epsilon_v + b'\theta] \tag{23b}$$

and thus using eqn (23a)

$$p = X\theta \tag{23c}$$

with

$$X = (\lambda + 2G)a_u - b'.$$

It remains to determine the displacement. Equations (20) and (23a) show that

$$\nabla^2\Omega = a_u\theta.$$

This equation can be integrated immediately and it is found that

$$\bar{\Omega} = A + \frac{B}{R} + a_u \frac{\kappa}{s}\bar{\theta}$$

whereupon taking into account the conditions of zero displacement on the surface of the sphere, it follows that:

$$\frac{\bar{U}_R}{R} = a_u \frac{s\bar{Q}}{4\pi KR}\bar{g} \tag{24}$$

where

$$\bar{g} = \frac{\kappa}{R^2 s^2}\left[1 - \frac{(1 + \xi R)}{(1 + \xi R_0)} e^{-\xi(R - R_0)}\right].$$

The stress distribution may be found from eqn (3). The only non zero stress components are

$$\bar{\sigma}_{RR} = 4Ga_u \frac{s\bar{Q}}{4\pi KR}\bar{g} \tag{25a}$$

$$\bar{\sigma}_{\phi\phi} = \sigma_{\psi\psi} = 2Ga_u \frac{s\bar{Q}}{4\pi KR}(\bar{f} - \bar{g}). \tag{25b}$$

When the intensity of the heat source does not vary with time the solution for an impermeable soil may be written

$$\theta = \frac{Q}{4\pi KR} f(\kappa, R, t)$$

$$p = \frac{Q}{4\pi KR} X\, f(\kappa, R, t)$$

$$\frac{U_R}{R} = a_u \frac{Q}{4\pi KR} g(\kappa, R, t) \tag{26}$$

$$\sigma_{RR} = 4a_u G \frac{Q}{4\pi KR} g(\kappa, R, t)$$

$$\sigma_{\gamma\gamma} = \sigma_{\phi\phi} = 2a_u G \frac{Q}{4\pi KR} [f(\kappa, R, t) - g(\kappa, R, t)]$$

where the function f was defined by eqn (22b) and where

$$g = \delta^2 + (\Delta R^2/2R^2 - \delta^2)\, erfc(\beta) + fR_0\Delta R/R^2 - \frac{\Delta R\delta}{R\sqrt{\pi}} e^{-\beta^2}$$

and

$$\delta^2 = \kappa t/R^2.$$

Equations (26) show that all field quantities may be expressed in terms of the two functions f, g. These functions are plotted in the form of isochrones in Figs. 3(a, b). Their variation with respect to time at particular locations are shown in Figs. 3(c, d).

3.3. *Solution for a permeable soil*

Next consider the case in which the soil is permeable. The temperature distribution is unaffected by the consolidation of the soil and so:

$$\bar{\theta} = \frac{s\bar{Q}}{4\pi KR} \bar{f}. \tag{28a}$$

Similarly the derivation of eqn (23b) remains unaffected by the value of permeability

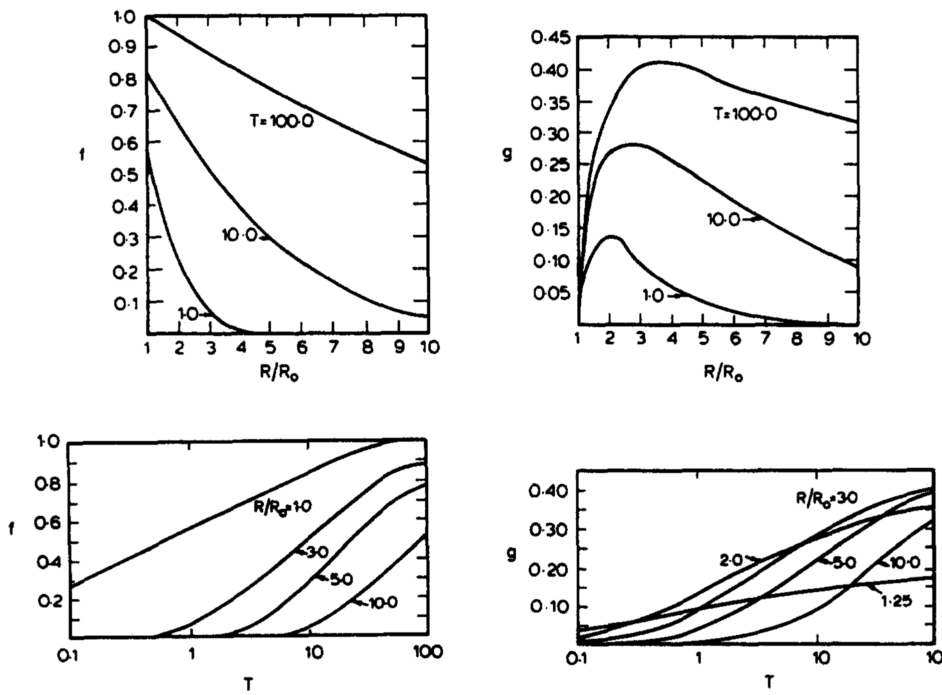


Fig. 3(a) Isochrones of the function f. (b) Isochrones of the function g. (c) Variation of function f with time. (d) Variation of function g with time.

and so:

$$\epsilon_v = -\frac{(p + b'\theta)}{\lambda + 2G}. \tag{28b}$$

If this relation is substituted into the transform of eqn (9) it is found, on taking due account of the initial condition, that the pore pressure satisfies the equation:

$$c\nabla^2\bar{p} = s(\bar{p} - X\bar{\theta}). \tag{29a}$$

We can see from this equation that the rate of temperature variation with time is acting as a source term. This term may be accounted for by seeking a particular solution proportional to the temperature it is the straight forward to show that

$$\bar{p} = \frac{A}{R}\,e^{-\eta(R-R_0)} + \frac{X\bar{\theta}}{1 - c/\kappa} \tag{29b}$$

where $\eta = \sqrt{(s/c)}$ denotes the square root having a positive real part and A is a constant to be determined.

The sphere $R = R_0$ is assumed to be impermeable and thus it is found that:

$$\bar{p} = \frac{X}{(1 - c/\kappa)} \cdot \frac{s\bar{Q}}{4\pi KR} \cdot [\bar{f}(s/\kappa) - \bar{f}(s/c)]. \tag{29c}$$

It then follows from eqn (28b) that

$$\bar{\epsilon}_v = -\left(\frac{\partial U_R}{\partial R} + \frac{U_R}{R}\right) = -a_u\frac{s\bar{Q}}{4\pi KR}\,\bar{f}^* \tag{30a}$$

where

$$\bar{f}^* = Y\bar{f}(s/\kappa) - Z\,\bar{f}(s/c)$$

and

$$Y = \frac{1}{\lambda + 2G}\left[\frac{X}{a_u(1 - c/\kappa)} + \frac{b'}{a_u}\right]$$

$$Z = \frac{1}{\lambda + 2G} \cdot \frac{X}{a_u(1 - c/\kappa)}.$$

It follows immediately that the solution of eqn (28a) is

$$\frac{\bar{U}_R}{R} = a_u\frac{s\bar{Q}}{4\pi KR}\,\bar{g}^* \tag{30b}$$

where $\bar{g}^* = Y\,\bar{g}(s/\kappa) - Z\,\bar{g}(s/c)$. *As before the stress distributions may be found from eqns (4), the only non-zero stress components being:*

$$\bar{\sigma}_{RR} = 4a_uG\,\frac{s\bar{Q}}{4\pi KR}\,\bar{g}^* \tag{30c}$$

$$\bar{\sigma}_{\phi\phi} = \bar{\sigma}_{\psi\psi} = 2a_uG\,\frac{s\bar{Q}}{4\pi KR}\,\bar{g}^*. \tag{30d}$$

Summarising the complete solution for a constant source is:

$$
\begin{aligned}
\theta &= \frac{Q}{4\pi KR} f(\kappa, R, t) \\
p &= \frac{X}{1 - c/\kappa} \cdot \frac{Q}{4\pi KR} \cdot [f(\kappa, R, t) - f(c, R, t)] \\
\frac{U_R}{R} &= a_u \frac{Q}{4\pi KR} g^*(t, R, R_0) \\
\sigma_{RR} &= 4a_u G \frac{Q}{4\pi KR} g^*(t, R, R_0) \\
\sigma_{\phi\phi} &= \sigma_{\psi\psi} = 2a_u G \frac{Q}{4\pi KR} [f^*(t, R, R_0) - g^*(t, R, R_0)]
\end{aligned}
\tag{31}
$$

where the functions f, g are defined by eqns (23b) and (27) respectively and

$$
\begin{aligned}
f^* &= Yf(\kappa, R, t) - Zf(c, R, t) \\
g^* &= Yg(\kappa, R, t) - Zf(c, R, t)
\end{aligned}
$$

clearly these equations reduce to the case of an impermeable soil, eqns (26), when $c \rightarrow 0$.

It can be seen from eqns (31) that all the field quantities can again be expressed in terms of the functions f and g. The distribution of temperature is uncoupled from the consolidation process and consequently is as shown in Figs. 2(a, b). If the soil were impermeable, it follows from eqn (26) that the pore pressure at the surface of the source would increase until it reached the maximum value.

$$p_N = [(\lambda + 2G)\, a_u - b']\frac{Q}{4\pi R_0} = X\theta_N.$$

If this value is used to normalise the pore pressure it is found from eqn (29)

$$\frac{p}{p_N} = \frac{f(\kappa, R, t) - f(c, R, t)}{1 - c/\kappa}.$$

This normalised pore pressure is independent of the elastic properties of the soil (except in so far as it effects the coefficient of consolidation) depending only upon position, time and the ratio of the coefficient of consolidation to the diffusivity, c/κ.

The variation of normalised pore pressure with time is shown in Figs. 4(a–c) for the values $c/\kappa = \frac{1}{3}$, 1, 3. The behaviour is as would be expected; at first the rise in temperature causes a corresponding rise in excess pore pressure, at the same time these pore pressures start to dissipate, and so the rate of increase of pore pressure decreases with time. After some time a maximum excess pore pressure is achieved, and thereafter the rate of dissipation of pore pressure exceeds the rate of generation due to temperature change and so ultimately all the excess pore pressures dissipate. Figures 4(a–c) show that the pore water pressure dissipation is most effective in reducing the severity of the rise of excess of pore pressure. Thus if $c/\kappa = 3$ the excess pressure rises to less than eight per cent of the value it would reach if no dissipation occurred, if $c/\kappa = 1$ the rise is a little under fourteen percent while even if $c/\kappa = \frac{1}{3}$, the rise is no more than twenty-three percent.

The variation of stress state is shown in Figs. 5(a–c) and Figs. 6(a–c) for the particular

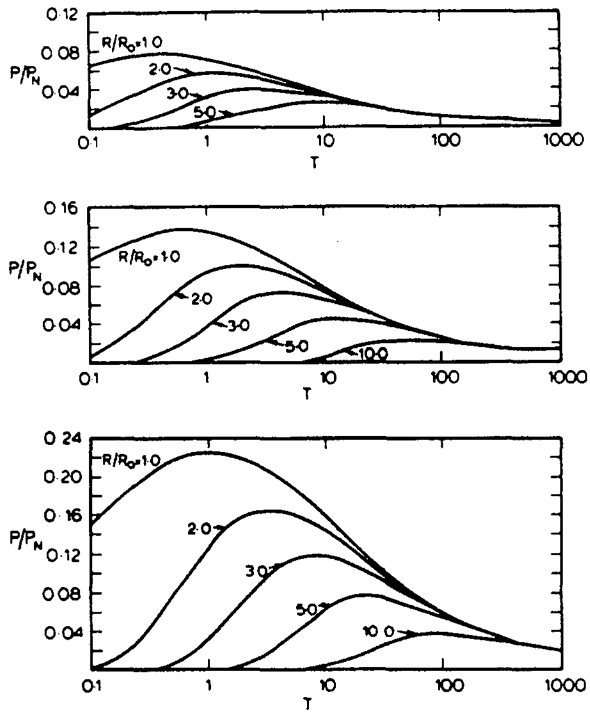


Fig. 4(a) Variation of pore pressure with time ($c/\kappa = 3$). (b) Variation of pore pressure with time ($c/\kappa = 1$). (c) Variation of pore pressure with time ($c/\kappa = \frac{1}{3}$).

case in which

$$\nu' = 0.4$$

$$a'/a_u = \tfrac{1}{4}$$

for the values $c/\kappa = 3, 1, \frac{1}{3}$.

These stresses have been normalised with respect to p_N. Two points emerge, first, the increase in radial stress is relatively small and compressive and its maximum value

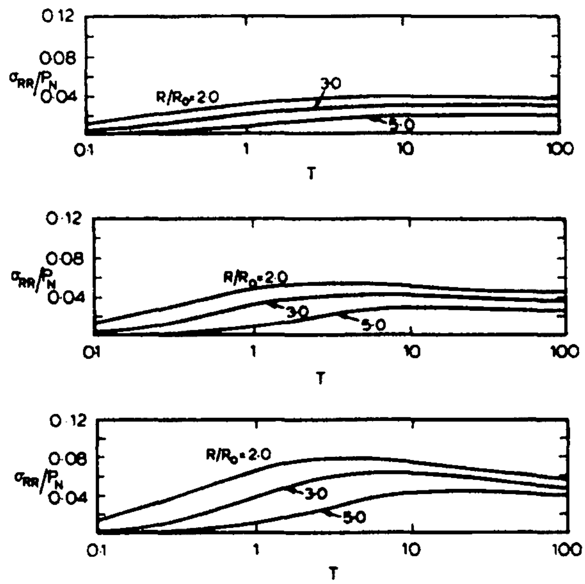


Fig. 5(a) Variation of radial stress with time ($c/\kappa = 3$). (b) Variation of radial stress with time ($c/\kappa = 1$). (c) Variation of radial stress with time ($c/\kappa = \frac{1}{3}$).

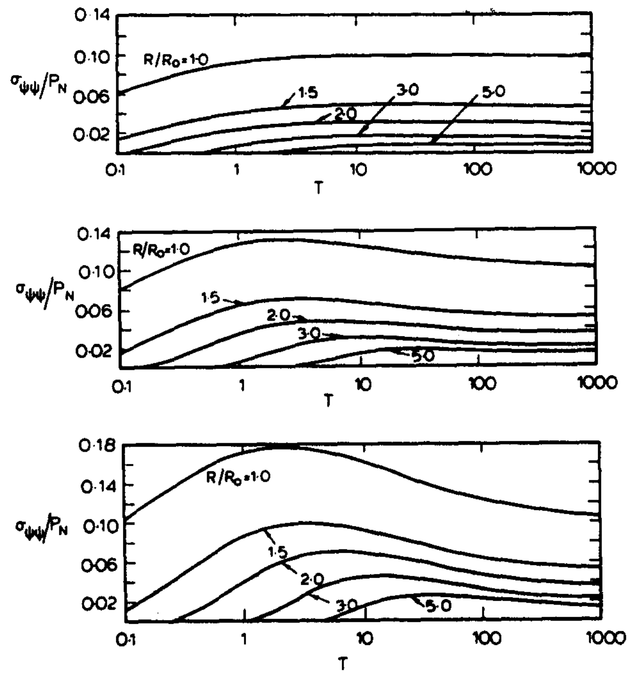


Fig. 6(a) Variation of hoop stress with time ($c/\kappa = 3$). (b) Variation of hoop stress with time ($c/\kappa = 1$). (c) Variation of hoop stress with time ($c/\kappa = \frac{1}{3}$).

increases as the soil becomes more impermeable. Second, the increase in circumferential stress is rather larger but it is also compressive. This means that the major reduction in effective stress is likely to occur in the radial direction.

3.4. *Solution for a point source*

The solution derived in the previous section can be used to determine the solution for a point source by allowing $R_0 \to 0$ and we find that:

$$\theta = \frac{Q}{4\pi KR} f_{ps}(\kappa, R, t)$$

$$p = \frac{X}{(1 - c/\kappa)} \frac{Q}{4\pi KR} [f_{ps}(\kappa, R, t) - f_{ps}(c, R, t)]$$

$$u_x = a_u x \frac{Q}{4\pi KR} g_{ps}^*$$

$$u_y = a_u y \frac{Q}{4\pi KR} g_{ps}^*$$

$$u_z = a_u z \frac{Q}{4\pi KR} g_{ps}^*$$

$$\sigma_{xx} = 2Ga_u \frac{Q}{4\pi KR} \left[f_{ps}^* - g_{ps}^* + \frac{x^2}{R^2} (3g_{ps}^* - f_{ps}^*) \right] \tag{33}$$

$$\sigma_{yy} = 2Ga_u \frac{Q}{4\pi KR} \left[f_{ps}^* - g_{ps}^* + \frac{y^2}{R^2} (3g_{ps}^* - f_{ps}^*) \right]$$

$$\sigma_{zz} = 2Ga_u \frac{Q}{4\pi KR} \left[f_{ps}^* - g_{ps}^* + \frac{z^2}{R^2} (3g_{ps}^* - f_{ps}^*) \right]$$

$$\sigma_{xz} = 2Ga_u \frac{Q}{4\pi KR} \frac{xz}{R^2} (3g_{ps}^* - f_{ps}^*)$$

$$\sigma_{yz} = 2Ga_u \frac{Q}{4\pi KR} \frac{yz}{R^2} (3g^*_{ps} - f^*_{ps})$$

$$\sigma_{xy} = 2Ga_u \frac{Q}{4\pi KR} \frac{xy}{R^2} (3g^*_{ps} - f^*_{ps})$$

where

$$f_{ps}(\kappa, R, t) = erfc\,(\beta)$$

$$g_{ps}(\kappa, R, t) = \delta^2 + (\tfrac{1}{2} - \delta^2)\, erfc\, \beta - \frac{\delta}{\sqrt{\pi}} e^{-\beta^2}$$

with

$$\beta^2 = R^2/4\kappa t$$

$$\delta^2 = \kappa t/R^2$$

and

$$f^*_{ps} = Yf_{ps}(\kappa, R, t) - Zf_{ps}(c, R, t)$$

$$g^*_{ps} = Yg_{ps}(\kappa, R, t) - Zg_{ps}(c, R, t).$$

4. CONCLUSIONS

A theory of the consolidation of soil for non isothermal conditions which takes account of the differential thermal expension of the pore water and soil skeleton and which is based on simple concepts of volume constraint and the effective stress principle has been developed.

This theory has been used to develop an analytic solution for an impermeable rigid, spherical, heat source and a point source surrounded by a thermo-elastic permeable soil. Examination of the solution shows that the rise in temperature causes the pore pressure to rise but that the excess pore pressures generated in this way are dissipated quickly and rise to only a small fraction of the value that they would achieve if the soil were completely impermeable. The stress changes around the sphere are in general small and compressive, the radial stress undergoing the smallest increase.

Acknowledgements—The authors gratefully acknowledge the value of discussion with Prof. A. N. Schofield and Dr. M. F. Randolph. The work presented in this paper was undertaken in connection with a contract held between the Department of Environment and Cambridge University.

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